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Impact of Children's math self-concept, math self-efficacy, math anxiety, and teacher competencies on math development

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HIGHLIGHTS

- Establishment of a solid math foundation in elementary school is critical for children's mathematical development.
- Attention to math self-concept of children should be part of teachers' professional development.
- Teachers' mathematical teaching knowledge contributes to children's mathematical development.
- Analyses of arithmetic fluency and mathematical problem-solving showed different outcomes.

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ABSTRACT

We examined to what extent children's development of arithmetic fluency and mathematical problem-solving was influenced by their math self-concept, math self-efficacy, and math anxiety but also teacher competence, specifically: actual teaching behavior, self-efficacy, and mathematical teaching knowledge. Participants were 610 children and 31 teachers of grade four. Multi-level analyses showed children's math self-concept to be a positive predictor of arithmetic fluency and actual teaching behavior to be a negative predictor. The development of mathematical problem-solving was predicted: positively by mathematical teaching knowledge; negatively by actual teaching behavior and teachers' self-efficacy; and not at all by the child factors of math self-concept, math self-efficacy, or math anxiety. Promoting the self-confidence of young children is essential for their mathematical development. More research into the relationship between teaching behaviors and children's math development is needed.

1. Introduction

The main goal of mathematics education today is to develop the knowledge and skills needed for later professional and personal lives (Organisation for Economic Co-operation and Development, 1999; Tout & Gal, 2015). Two essential subdomains are arithmetic fluency (i.e., the ability to add, subtract, multiply, and divide fast and accurately) and mathematical problem-solving (i.e., solving problems using mathematical notation, text, and/or pictures) (National Research Council, 2001; Powell, Fuchs, & Fuchs, 2013). Math is known to be hard for some children due to such factors as low math self-esteem and no appropriate math education (Mazzocco, 2007).

To understand the development of children's math skill, research has paid more attention to cognitive, information-processing, and neuropsychological factors and less attention to child self-perceptions and beliefs about math skill. However, children's math self-concept (Bong & Clark, 1999; Timmerman, Toll, & Van Luit, 2017), math self-efficacy (Bandura, 1997; Joët, Usher, & Bressoux, 2011; Pajares & Miller, 1994), and math anxiety (Ashcraft & Moore, 2009; Ramirez, Chang, Maloney, Levine, &

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Beilock, 2016) have been shown to significantly correlate with math achievement. In general, better math skill positively correlates with math self-concept and math self-efficacy while poorer math skill negatively correlates with math anxiety. Similarly, children's math development has been shown to be significantly associated with the observed math teaching behavior of teachers (Muijs & Reynolds, 2000, 2002; Stronge, Ward, & Grant, 2011), mathematical teaching knowledge (Baumert et al., 2010; Hill, Rowan, & Ball, 2005), and teachers' self-efficacy (Klassen et al., 2009; Tella, 2008).

Research has yet to consider the roles of *both* child and teacher factors together for understanding children's math development. In addition, arithmetic fluency and mathematical problem-solving are not distinguished clearly in most research despite the involvement of different underlying skills. In the current study, we therefore investigated the influences of two sets of factors on the development of the arithmetic fluency and mathematical problem-solving abilities. We examined, in particular: 1) the math self-concepts, math self-efficacy, and math anxiety of fourth grade children and 2) the actual math teaching behavior, teaching knowledge, and teaching self-efficacy of their teachers.

1.1. Math development

During early elementary school, children are expected to develop an understanding of numbers, counting, and simple arithmetic (Geary, 2003). With increasing arithmetic speed and accuracy, a solid foundation is assumed to be laid for the development of more advanced mathematical problem-solving abilities (Gersten, Jordan, & Flojo, 2005). Geary (2004) has provided a theoretical framework in which math development is assumed to relate to the combined functioning of the visuospatial and language systems, the central executive functioning of the brain, conceptual development, and procedural knowledge (e.g., knowledge of rules and algorithms). Knowledge of basic arithmetic combinations is stored in long-term memory and easily retrieved for the solution of math problems using short-term memory information (Baddeley, 2000). The development of arithmetic fluency and mathematical problem-solving can thus be seen as distinct aspects of children's math development (Fuchs et al., 2008).

Arithmetic fluency is the ability to add, subtract, multiply, and divide with basic number combinations accurately and quickly. The development of arithmetic fluency starts with the onset of formal math education. As part of early elementary education (children aged 6–8 years), considerable attention is paid to the promotion of arithmetic knowledge and fluency. The speed and accuracy of children's performance on arithmetic fact problems increases between the first and seventh grades (Ostad, 2000) with attention and processing speed identified as key factors (Fuchs et al., 2008). And the later math development of children who have difficulties retrieving basic arithmetic facts from long-term memory has been shown to be hampered (Duncan et al., 2007; Geary, 2004; Geary & Hoard, 2005).

Mathematical problem-solving can be defined as the ability to apply mathematical knowledge and skills to solve actual or imagined "real life" imaginable problems using mathematical notation, text, and/or pictures. Mathematical problem-solving is taught in mainly the upper grades of elementary school. From about fourth grade (children aged 8–10 years), the focus of mathematics education shifts to advanced mathematics (e.g., fractions, proportions) and the abstractness and complexity of math tasks increases. Mathematical problem-solving requires children to be able to read the problem, distinguish relevant from irrelevant information, identify key words, derive underlying numerical relationships, select and apply required operations and algorithms, and manipulate numbers procedurally (Fuchs et al., 2008; Goldin, 1998; Kintsch & Greeno, 1985). The brain's central executive system of working memory plays an important role in the integration of information for the solution of mathematical problems and has thus been found to be an important predictor of developing mathematical problem-solving ability (Swanson & Beebe-Frankenberger, 2004).

Several longitudinal studies have shown strong associations between early and later math achievement (Byrnes & Wasik, 2009; Duncan et al., 2007; Watts, Duncan, Siegler, & Davis-Kean, 2014). And the developments of both arithmetic fluency and mathematical problem-solving have been shown to be highly stable with early math skill critical for the development of later math skill (Fuchs et al., 2006; Watts et al., 2014).

There is nevertheless evidence that *additional* child and teacher factors are crucial for the development of math skill.

1.2. Roles of children's math self-concept, math self-efficacy, and math anxiety

As already mentioned, children's math development depends on several factors with cognitive factors receiving the most attention in previous research. Math development have also been shown to relate to children's math self-beliefs (Bandura, 1997; Pajares & Miller, 1994). In the first years of elementary school, children have positive and even at times unrealistic perceptions of their abilities. These early self-beliefs are relatively unstable (Wigfield & Eccles, 2000). By the age of seven/eight years, children have become more sensitive to performance feedback and their self-perceptions become more realistic and stable (Dweck, 2002).

Three aspects of math self-belief have been distinguished to date: math self-concept, math self-efficacy, and math anxiety. *Math self-concept* subsumes beliefs about self-worth associated with math competence. In general, self-concept is less specific than self-efficacy (Bong & Clark, 1999). *Math self-efficacy* is a judgment of one's capacity to perform domain-specific tasks— for example — solve word math problems or fact problems and succeed (Bandura, 1997). A child may have a generally positive, math self-concept but hold quite different beliefs about specific math tasks (i.e., negative self-efficacy at times). *Math anxiety* is a negative emotional response to numbers and/or math-related situations (Suárez-Pellicioni, Núñez-Peña, & Colomé, 2016).

Positive correlations have generally been found between math self-concept and math achievement (McWilliams, Nier, & Singer, 2013; Möller, Pohlmann, Köller, & Marsh, 2009). Viljaranta, Tolvanen, Aunola, and Nurmi (2014) did not, however, find math self-concept to *predict* subsequent math achievement. Timmerman et al. (2017) found positive correlations between math self-concept and both arithmetic fluency and mathematical problem-solving in adolescents. Previous experiences with mathematical problem-solving can obviously contribute to math self-concept (Elbaum & Vaughn, 2001) while math self-concept can conversely influence math performance (Marsh, Trautwein, Lüdtke, Köller, & Baumert, 2005). By grade four, reciprocal associations have indeed been found with children's self-concept significantly influencing their math achievement and vice versa (Weidinger, Steinmayr, & Spinath, 2018).

Children's *experience* with mathematics tasks in the past has been shown to be most influential for math self-efficacy (Usher & Pajares, 2008, 2009). In addition, the receipt of efficacy-related information including positive social messages about math performance and evaluative feedback from teachers but also experienced emotional states and physiological reactions have been shown to significantly influence math self-efficacy (Bandura, 1997; Joët et al., 2011). Furthermore, Pietsch, Walker, and Chapman (2003) have shown math self-efficacy to correlate more strongly with math achievement than math self-concept does. Pajares and Kranzler (1995) showed math self-efficacy, moreover, to be *predictive* of math achievement in general and mathematical problem-solving in particular.

Lee (2009) found clear cross-cultural differences when she examined all three aspects of math self-belief in conjunction with the math achievement of 276,165 children aged 15 years using PISA 2003 questionnaire data from 41 countries. The strongest associations between math self-concept and math achievement were found in Western European countries. The strongest associations between math self-efficacy and math achievement were found in Asian and Eastern European countries. The associations between math anxiety and math achievement were stronger in Western and Eastern European countries than in Asian countries. And some of the Western European countries, including the Netherlands, showed particularly low levels of math anxiety.

Inconsistent findings have nevertheless been found for math anxiety in relation to young children's math achievement (Dowker, Sarkar, & Looi, 2016). Math anxiety was found to negatively correlate with math achievement due to avoidance of mathematics, the suppression of cognitive processing by anxiety, and/or the roles of social factors (e.g., teachers' and parents' own math anxiety) (Ashcraft, 2002; Ashcraft & Kirk, 2001; Maloney & Beilock, 2012). Math anxiety has been shown to interfere with working memory and thereby have a strong effect on math achievement (Ashcraft & Kirk, 2001). Thoughts about how badly one is doing or may do (i.e., aspects of math anxiety) can distract attention from the task at hand and overload working memory at the same time. Timmerman et al. (2017) nevertheless found no significant associations between math anxiety and arithmetic fluency. With regard to mathematical problem-solving, however, Ramirez et al. (2016) found math anxiety to indeed be a negative predictor of the adoption of advanced problem-solving strategies and a positive predictor of lower achievement for mathematical problem-solving. They also found both the math anxiety and mathematical problem-solving strategies to be strongest for the children with the greatest working memory capacity in the same study. In sum, mathematical difficulties and experiences of failure during the early school years can elicit and increase math anxiety. As a consequence, children may avoid further learning in the domain of mathematics, acquire increasingly more negative experiences with math, and become more anxious with regard to math. A vicious cycle thus emerges.

Most of the aforementioned research was cross-sectional, which precludes the drawing of conclusions about causal relations between — on the one hand — math self-concept, math self-efficacy, and math anxiety and — on the other hand — math achievement. Most of the relevant studies concerned only high school students, moreover. And most of the studies considered only one aspect of self-belief (i.e., math self-concept or math self-efficacy or math anxiety) in connection with math achievement.

1.3. Role of teacher competencies

As might be expected, teacher characteristics and competencies can influence student math achievement. In research, three specific teacher competencies have been examined in relation to student math achievement: the actual behavior of the teacher during math lessons (e.g., Stronge et al., 2011), teacher's mathematical teaching knowledge (e.g., Campbell et al., 2014), and teacher's self-efficacy with respect to the teaching of math (e.g., Klassen et al., 2009).

When Van de Grift (2007) observed 854 math lessons of teachers of nine year old children, the following teacher variables were found to play a critical role in students' math achievement: a safe and stimulating learning climate, clear instruction, adapted teaching, type of teaching and learning strategies (e.g., model, explain, scaffold), and efficient classroom management. When Stronge et al. (2011) compared outcomes of observed lessons with data on teacher effectiveness, they found classroom management but also the relationships with students to correlate most strongly with math achievement. In contrast, Blazar (2015) found no associations of classroom climate and classroom management with math achievement. He found instead that inquiry-orientated instruction positively predicted student achievement. Reynolds and Muijs (1999) found that both whole-class interactive and collaborative group-based teaching positively influenced achievement for a range of math skills. In another study, Muijs and Reynolds (2002) found effective teacher behavior (e.g., interactive math teaching, direct instruction), positive self-efficacy beliefs, and good subject knowledge to significantly correlate with students' math achievement. Noteworthy, they found constructivist math teaching to negatively correlate with math development. In other research, Wenglinsky (2000) concluded that the use of hands-on learning activities to illustrate mathematical concepts and stimulate higher-order thinking skills can promote math achievement. Hiebert and Grouws (2007) concluded, based on their review, that teacher behavior is effective if teachers are explicit about learning goals, make their teaching behavior dependent on the math learning goal, and foster engagement particularly on the part of children who are struggling with mathematics. Teaching behavior that facilitates the development of understanding of math concepts and makes the connections between ideas, facts, and procedures sufficiently explicit was found to be important for children's mathematical development (e.g., interactive instruction, think-stimulating activities, comparison of solution strategies, critical thinking). A meta-analysis focusing on teaching factors related to student outcomes (Kyriakides, Christoforou, & Charalambous, 2013) showed student achievement to not be associated with a single teaching approach (e.g., direct vs. constructivist instruction); making well-considered choices and adoption of elements of different approaches were found to be crucial instead.

In observational research specifically concerned with the influences of teacher behavior on arithmetic fluency, Kling and Bay-Williams (2014) found giving children opportunities to notice relationships, adopt strategies, and practice with these strategies to promote arithmetic fluency. Muijs and Reynolds (2000) found active, whole-class teaching that clearly involves students to be associated with better achievement in arithmetic fluency. Teacher behaviors considered together, moreover, explained the basic math achievement of students while individual teacher behaviors did not (e.g., organization, time spent on interactive teaching).

Regarding mathematical problem-solving, instruction focused on strategies for solving different types of problems and direct teaching of higher-level cognitive strategies were shown to improve achievement (Verschaffel et al., 1999; Wenglinsky, 2000).

Mathematical teaching knowledge concerns knowledge of required math concepts, possible misconceptions on the part of students, effective instructional strategies, and various representations. Mathematical teaching knowledge is subject-specific and content knowledge forms a necessary prerequisite for the connection of pedagogy with context (Depaepe, Verschaffel, & Kelchtermans, 2013). Hill et al. (2005) found teachers' mathematical teaching knowledge to positively predict gains in student math achievement during the first and third grades. Similarly, Campbell et al. (2014) found teachers' mathematical teaching knowledge to directly and positively relate to children's math achievement in grades four through eight. In a study by Muijs and Reynolds (2002), in which they collected data indirectly through a self-perception questionnaire, mathematical content knowledge correlated strongly with teachers' self-efficacy beliefs and only to a lesser extent with students' math development.

Teaching self-efficacy refers to teachers' perceptions of their capacity to promote student learning, achievement, and engagement (Bandura, 1993, 1997; Tschannen-Moran & Woolfolk Hoy, 2001). In a review by Klassen, Tze, Betts, and Gordon (2011), ambiguous results were found for associations between teachers' self-efficacy and general student achievement. In other research, however, Tella (2008) found teachers' self-efficacy to contribute significantly to children's math achievement. Ashton and Webb (1986) also found a positive correlation between teachers' self-efficacy and student math achievement.

1.4. The present study

Despite the widespread availability of research addressing the impact of teacher-related factors on student achievement, relatively little is known about the influence of *specific* teacher competencies on student *math* performance. Research that takes a) the actual behavior of teachers, b) their mathematical knowledge, and c) their math teaching self-efficacy into account is quite scarce. Basic arithmetic fluency is rarely distinguished from later mathematical problem-solving, moreover. And consideration of the aforementioned factors *together* in a single study has yet to be done. In the present study, we thus examined the influences of specific teacher competencies *together* with children's math self-concepts, math self-efficacy, and math anxiety on children's math development over time. A longitudinal design was adopted to allow us to monitor children's math development from the start to the end of the fourth grade.

The general research question was: How do a) children's math self-concept, math self-efficacy, and math anxiety, b) teacher competencies, and c) combinations of these child and teacher factors predict the development of children's arithmetic fluency and mathematical problem-solving during the course of the fourth grade?

We expected, even after control for the children's entrance-level math abilities, both the child and teacher factors to make unique contributions to the development of both arithmetic fluency and mathematical problem-solving.

2.0. Method

2.1. Participants and study context

Participants were recruited via social media (Twitter) and letters to both elementary school principals and fourth grade teachers (contact information gathered via public websites for schools). Two-thirds of those approached responded to the open invitation, which included information on the aims of the study, what was expected of the participants, and what the participants could expect of the researchers. In the end, 31 teachers agreed to participate and the study was conducted during the 2016-17 school year in the Netherlands.

The teachers worked with 610 children at 27 elementary schools located in different parts of the Netherlands. The sizes of the schools varied: 6% had fewer than 100 children (small); 66% had between 100 and 400 children (medium); and 28% had more than 400 children (large). The composition of the classes varied: 66% homogeneous (all fourth grade); 34% heterogeneous (combination of two grades in one class). The mean age of the teachers was 38; 1 (years; months) (range of 24–60 years) with 16% male and 84% female. The majority of the teachers had a bachelor's degree in education (66%); 28% had additional graduate training; and 6% had a Master's degree in education. The teachers had an average of 11.9 years of experience ($SD = 8.7$) (range of 2–39 years).

Of the 610 children, 53% was male and 47% female. The age of the fourth graders ranged from 8; 2 to 10; 10 with a mean of 9; 2 ($SD = 0.31$). The wide spread in age was due to either having skipped a year or having stayed behind a year. The home language for 88.5% of the children was Dutch.

The children's nonverbal reasoning was tested using the Raven's Standard Progressive Matrices (SPM). It was checked that none of the children scored two or more standard deviations below the mean (Raven, 2000; Raven, Raven, & Court, 1998). None of the children did. The mean nonverbal reasoning score for the children was 36.64 ($SD = 7.43$), Skewness −0.86, Kurtosis 1.51.

2.2. Measurement instruments

2.2.1. Math achievement

Children's math achievement was measured using two instruments: a test of arithmetic fluency (addition, subtraction, multiplication, and division) and a test of advanced mathematical problem-solving (fact and word problems).

Arithmetic fluency. The Speeded Arithmetic Test (TTA; De Vos, 2010) is a standardized paper-and-pencil test frequently used in Dutch and Flemish education to measure speeded arithmetic skill (arithmetic fluency). The test consists of four categories of 50 fact problems: addition (tasks with a difficulty level varying from $6 + 0$ to $29 + 28$), subtraction (difficulty level varying from 4 -2 to 84–38), multiplication (difficulty level varying from 4×1 to 7×9), and division (difficulty level varying from 6 : 2 to 72 : 9). Children are given 2 min per category of problems. Each correct answer yields one point, for 50 possible points per category and a total possible score of 200. The total score was used in the analyses. The reliability and validity of such testing has been found to be good ($\alpha = 0.88$; De Vos, 2010).

Mathematical Problem-solving. Children's math achievement was measured using the criterion-based mathematics tests (Cito; Janssen, Scheltens, & Kramer, 2005), which are standardized Dutch national test commonly administered at the middle and end of each school year to monitor children's progress. The test consists of a mixture of math problems in several domains presented in varied ways: only using mathematical notation or combinations of text, mathematical tasks related pictures, and mathematical notation as used in regular curricula (e.g., *There are 24 boxes in a warehouse. Each box contains 8 cans of soup. How many cans of soup are there?*). The following domains are covered: 1) numbers, number relations, and operations (addition, subtraction, multiplication, and division), 2) proportions and fractions, and 3) measurement and geometry. The reliability coefficients for the tests have been found to range from 0.91 to 0.97 (Janssen, Verhelst, Engelen, & Scheltens, 2010).

2.2.2. Child factors

The *math self-concept*, *math self-efficacy*, and *math anxiety* of the students were measured using the Mathematics Motivation Questionnaire for Children (MMQC; Prast, Van de Weijer-Bergsma, Kroesbergen, & Van Luit, 2012). The questionnaire consists of five scales: math self-efficacy (6 items), math self-concept (6 items), math task value (7 items), math lack of challenge (6 items), and math anxiety (5 items). Items are rated along a four-point scale: 1 = NO! (strongly disagree), 2 = no (disagree), 3 = yes (agree), 4 = YES! (strongly agree). A sample item from the math self-concept scale is "Are you good in mathematics?". A sample item from the math self-efficacy scale is "When the teacher explains the first math problem, are you capable of solving the next math problem by yourself?". A sample item from the math anxiety scale is "Are you afraid to make mistakes during the math lesson?". These three scales were used in the present study and their internal consistency was found to be good (self-concept $\alpha = 0.91$; self-efficacy $\alpha = 0.81$; math anxiety $\alpha = 0.79$).

2.2.3. Teacher competencies

Actual Teaching Behavior in Math Lessons. The actual teaching behavior of the teachers in their math lessons was measured using the International Comparative Analysis of Learning and Teaching (ICALT), an observation instrument (Van de Grift, 2007). The ICALT, consisting of seven scales, covers many aspects of teaching behavior and is not math-specific. For purposes of the present study, the instrument was therefore supplemented with an eighth scale specifically addressing the teaching of mathematics. The ICALT itself involves 32 items addressing six aspects of teaching behavior ranging from lower order teaching behavior to higher order teaching behavior (Van der Lans, Van de Grift, & Van Veen, 2015, 2018): a) safe and stimulating learning climate, b) efficient classroom management, c) quality of instruction, d) student activation, e) teaching of learning strategies, and f) differentiation/adaptation of lesson content to meet children's learning needs. The seventh scale addresses student involvement. The eighth scale addressed math-specific teaching strategies using the following 8 items: a) informal manipulation, b) representations of real objects and situations, c) abstract mental representations (models and diagrams), d) abstract concepts/mental operations, e) connecting these four levels and using these appropriate to the goal of the lesson, pay attention to f) planning, g) solving processes, and h) metacognitive skills. All of the scales used in the present study were found to have reliable Cronbach's alphas. The internal consistency of the ICALT has been found in the past to be good ($\alpha = 0.82$). The internal consistency of the ICALT with the supplemental scales (ICALT + S) used in the present study was similarly found to be good ($\alpha = 0.85$).

Mathematical Teaching Knowledge. Teachers' mathematical teaching knowledge was self-assessed using a questionnaire specifically developed for the present study: the Teachers' Sense of Mathematical Knowledge for Teaching Questionnaire (SMKTQ; Kaskens, Segers, Goei, Verhoeven, & Van Luit, 2016). Composed of three parts and 38 questions, the following are assessed: a) mathematical skill in the domains of numbers, number relations and operations, proportions and fractions, measurement and geometry (*Subject Matter Knowledge*); b) ability to follow and analyze children's thinking including recognition of errors and responding to these (*Pedagogical Content Knowledge*); and c) selection and use of models and representations for different domains of math, use of real-world contexts, and knowledge of the metric system (*Specialized Content Knowledge*). Teachers responded to items along a four-point scale ranging from 1 (= *to a very small extent*) to 4 (= *to a very large extent*). The internal consistency of the SMKTQ was found to be good ($\alpha = 0.93$).

Teachers' Self-efficacy. The Dutch online version (Goei & Schipper, 2016) of the long form of the Teachers' Sense of Self Efficacy Scale (TSES; Tschannen-Moran & Woolfolk Hoy, 2001) was used to measure teachers' self-efficacy with respect to the teaching of math. The questionnaire contains 24 items equally divided across three subscales: a) efficacy for student engagement (e.g., *How much can you do to help students think critically?*), b) efficacy for instructional strategies (e.g., *How well can you respond to difficult questions from your students?*), and c) efficacy for classroom management (e.g., *How much can you do to get children to follow classroom rules?*). The teachers responded along a nine-point scale ranging from 1 (= *not at all*) to 9 (= *a great deal*). Reliability was found to be good in the present study: the Cronbach's alphas for the three subscales were 0.74, 0.81, and 0.82, respectively.

2.3. Procedure

After recruitment of participants, an information meeting was held in two different regions of the Netherlands. During the meeting, the teachers were given written information about the study and a factsheet about the methods of data collection to be used. The teachers consented via e-mail for subsequent observation and video-recording of a regular math lesson taught by them on the topic of fractions or ratios.

The parents of students were provided written information about the study by the teacher. Their written consent for participation of their child in the study was obtained prior to data collection. The sample was treated in accordance with institutional guidelines as well as with APA ethical standards.

2.4. Data collection

As part of a larger longitudinal research project, data of children and teachers were obtained on two measurement occasions: at the start of the school year (in May–June) (= T1) and the end of the school year (in September–October) (= T2).

Teachers. The SMKTQ and TSES were sent to the 31 participating teachers using the web-based questionnaire services of Formdesk (SMKTQ and TSES). An email was sent with a direct link to the Formdesk questionnaires and the teachers were asked to complete the two questionnaires. This was done at the beginning and the end of the school year with two reminders sent on each occasion. Response rate was 100%; all collected data from the 31 teachers was thus included in subsequent analyses.

For purposes of observation (and video recording), the teachers were asked to teach as normal as possible in order to provide representative data. It was agreed that the topic of the lesson would be in the domain of fractions or proportions. In accordance with the procedure of Van de Grift, Helms-Lorenz, and Maulana (2014), the ICALT + S observations were conducted by two trained observers. The training consisted of an explanation of the observation instrument, group discussions, and the rating of three video-recorded sample lessons. For each sample lesson, observers scored the 40 items from the ICALT + S along a four-point Likert scale ranging from 1 (= *predominantly weak*) to 4 (= *predominantly strong*). Observers who met the consensus norm of 0.70 or higher were judged to be sufficiently qualified. All of the observed math lessons were also video recorded. The inter-rater reliability for live scoring was good (0.86). The first author conducted 65% of the observations; a fellow observer conducted the remaining observations.

On the same day as the ICALT observation of the teacher, data were collected from the children.

Children. The MMQC, TTA, and RAVEN were conducted using paper and pencil in the class, with one examiner giving instructions. The teacher remained in the classroom. Children were positioned in a test setup so that they were not able to copy from one another. The examiner remained in the classroom at all times to answer any questions. The procedure lasted approximately 65 min (excluding breaks, which were arranged for the children and taken periodically).

The Cito math achievement data were obtained from the participating teachers, with parental consent.

The participating teachers were debriefed after measurement and thus informed of results. Due to illness or other reasons for school absence, relocation to a new school during the school year, or incomplete test responding, the number of data points for the children per test varied from 525 to 610.

2.5. Data analyses

The data and descriptive statistics for all of the measures were first screened for potential errors and outliers. Three separate multilevel models were then operationalized to examine: a) the extent to which child-related factors influence their math development (model 1); b) the extent to which teacher-related factors influence math development (model 2); and c) the extent to which child- and teacher-related factors considered together influence math development (model 3). The models were structured incrementally. And in each of the three models, Arithmetic Fluency (AF) and mathematical Problem-Solving (PS) were distinguished as individual measures of math achievement.

In a two-level hierarchical structure, arithmetic fluency (AF) ($N = 525$) (T2) and mathematical problem-solving (PS) ($N = 576$) (T2) were nested within teacher/class ($N = 31$). Given the nested structure of the data (i.e., children within classes) and the sample size of 31 teachers/classes, we therefore decided to first investigate whether multilevel modelling was actually needed. The intra-class correlation (ICC) and the design effect (Deff) were computed with the mixed model procedure of SPSS 25. The sample sizes at the classroom level were relatively small, which meant that restricted maximum likelihood (RML) estimation was employed (Hox, 2010). For completeness, maximum likelihood (ML) estimation and restricted maximum likelihood (REML) estimation were compared, but the ICC and Deff were equal.

The multilevel models were built according to the procedures of Heck, Thomas, and Tabata (2014) and Peugh (2010). All of the analyses started from the unconditional models in which the mean levels of the dependent variables were estimated while taking into account the variances at the levels of child and teacher/classroom. The unconditional "null" models were used to test the multilevel structure of the data. Subsequent models were then built including all predictors ("full" model). Nonsignificant predictors were next removed from the models to create the final "restricted" models. The fit indices for the final models were compared to those for the unconditional models to determine model improvement. A deviance statistic (-2 log likelihood) was calculated to decide if model fit improved. The deviance statistic had a large sample chi-square distribution, with degrees of freedom equal to the between-model difference in the number of parameters estimated. The significance of the improvement in model fit was tested using a χ^2 difference test. For math achievement AF, the ICC was 0.10 and Deff 2.51. For math achievement PS, ICC was 0.255 and Deff 5.48. Because the ICCs >0 and the Deffs >2 (Peugh, 2010), multilevel linear models were tested in all of the subsequent analyses. Continuous predictor variables were grand mean centered.

3.0. Results

3.1. Descriptive statistics

The means, standard deviations, and ranges for the different measures are presented in Table 1. All variables were normally distributed, with skewness and kurtosis within the normal ranges (Tabachnick & Fidell, 2013). Before turning to the research question, we also established that the math achievement of the children indeed increased during the school year. Paired samples t tests showed higher scores at the end of the school year than at the beginning for the two measures of math achievement: (arithmetic fluency, t (519) $= 19.92$, $p < .001$, $d = 0.57$; problem-solving t (552) $= 20.18$, $p < .001$, $d = 0.77$).

Pearson's correlation coefficients were next computed between the various child and teacher factors (Table 2). All of the child measures correlated significantly with the child math achievement measures. In addition: actual teaching behavior correlated significantly with mathematical problem-solving at the end of the year (T2); mathematical teaching knowledge correlated significantly with both arithmetic fluency at the start of the year (T1) and mathematical problem-solving at the start of the year (T1); and the math teachers' self-efficacy correlated significantly with their actual teaching behavior, on the one hand, and their mathematical teaching knowledge, on the other hand.

3.2. Children's math self-concept, math self-efficacy, and math anxiety as predictors of math development

The first part of our research question concerns the extent to which the children's math development during fourth grade was predicted by their math self-concept, math self-efficacy, and math anxiety when measured at the start of the school year. To answer this question, multi-level analyses were computed separately for the children's Arithmetic Fluency (AF) and mathematical Problem-Solving abilities (PS).

For Arithmetic Fluency (AF), the unconditional model with AF (T2) as dependent variable showed the level 1 math achievement scores of the children to vary significantly. To create the full model, all of the predictors were added into the unconditional model as fixed effects: that is, prior AF achievement (i.e., the initial measurement of AF, T1), math self-concept, math self-efficacy, and math anxiety. The full model showed a deviance statistic (-2 log likelihood) of 4458.58, indicating that the fit was significantly better than that provided by the unconditional model (i.e., the model not including these predictors) ($\beta = 752.25$, $p < .001$). Prior achievement ($M = 0.77$, $SD = 0.28$, $p < .001$) and math self-concept ($M = 1.64$, $SD = 0.53$, $p < .01$) were significant predictors of AF (T2). Math self-efficacy ($M = -0.88$, $SD = 0.59$, $p = .14$) and math anxiety ($M = 0.15$, $SD = 0.25$, $p = .54$) were not. This level-1 full model explained 11% of the total variance in the children's AF, T2 (ICC $= 0.11$).

We next computed the restricted model by removing all nonsignificant predictors from the model (in this case: math self-efficacy and math anxiety). The level-1 restricted model did not provide a better fit for the data relative to the level-1 full model ($\beta = 44.32$, $SD = 3.08$, $p < .001$; prior AF achievement $M = 0.77$, $SD = 0.03$, $p < .001$; math self-concept $M = 0.87$, $SD = 0.24$, $p < .001$; ICC $= 0.11$); the outcomes for the restricted model are therefore not presented in Table 3. In order to control for nesting within teacher/class, we finally computed the random effects for level 2 (class). Measures of children's development AF were thus corrected for the possible influences of teacher/class. Prior achievement ($M = 0.78$, $SD = 0.03$, $p < .001$) and math self-concept ($M = 1.71$, $SD = 0.52$, $p < .001$) continued to be significant predictors. This model explained 14% of the total variance in the children's AF, T2 (ICC $= 0.14$).

The same analyses were conducted for the children's mathematical problem-solving (PS). The coefficients and ICCs for the different models are presented in Table 3. The unconditional model showed the level-1 math achievement (PS) scores of the children to vary significantly. When all of the predictor measures were added to the unconditional model as fixed effects to create a full model, a deviance statistic (-2 log likelihood) of 4588.85 was found, showing the fit of the full model to be significantly better than the fit of the unconditional model ($\beta = 811.29$, $p < .001$). Prior PS achievement (i.e., the initial measurement of PS, T1) ($M = 0.74$, $SD = 0.03$, $p < .001$) significantly predicted PS achievement, T2. The children's math self-concept ($M = 0.28$, $SD = 0.46$, $p = .55$), math self-efficacy ($M = 0.32$, $SD = 0.51$, $p = .54$), and math anxiety ($M = 0.17$, $SD = 0.21$, $p = .42$) were not found to be significant predictors. This level-1 full model explained 22% of the total variance in the children's PS, T2 (ICC $= 0.22$).

When the restricted model was created by removing all nonsignificant predictors (i.e., math self-concept, math self-efficacy, and math anxiety), a better fit was not obtained ($\beta = 69.29$, $SD = 5.93$, $p < .001$; prior PS achievement $M = 0.77$,

Table 1
Measures of child and teacher factors.

Child					
	N	*M* (*SD*)	Range	Skewness	Kurtosis
Arithmetic fluency T1	610	105.22 (35.72)	(9–185)	0.19	−0.65
Arithmetic fluency T2	525	125.81 (34.72)	(34–196)	−0.11	−0.62
Math. problem-solving T1	586	217.43 (26.08)	(131–321)	−0.14	0.52
Math. problem-solving T2	576	237.77 (26.35)	(84–319)	−0.57	1.91
Math self-concept T1	605	20.40 (5.37)	(7–30)	−0.44	−0.60
Math self-efficacy T1	605	17.79 (3.45)	(7–28)	−0.35	0.22
Math anxiety T1	605	11.41 (4.25)	(6–24)	0.85	0.16
Teacher					
Actual teaching behavior	31	2.86 (0.25)	(2.39–3.38)	−0.01	−0.81
Math teachers' self-efficacy	31	7.08 (0.44)	(6.13–7.96)	−0.31	−0.71
Math. teaching knowledge	31	3.15 (0.30)	(2.47–3.87)	−0.17	0.13

Table 2
Correlations between child and teacher factors.

Measure	1	2	3	4	5	6	7	8	9	10
1. Arithmetic Fluency T1	–									
2. Arithmetic Fluency T2	833[a]	–								
3. Math. Problem-Solving T1	.529[a]	529[a]	–							
4. Math. Problem-Solving T2	467[a]	463[a]	773[a]	–						
5. Math self-concept	534[a]	512[a]	.552[a]	.473[a]	–					
6. Math self-efficacy	.465[a]	428[a]	.440[a]	.394[a]	910[a]	–				
7. Math anxiety	-.300[a]	-.303[a]	-.349[a]	-.265[a]	-.598[a]	-.563[a]	–			
8. Actual teaching behavior	.055	-.038	-.047	-.152[a]	.024	.033	.026	–		
9. Math. teaching knowledge	-.083[b]	-.064	-.092[b]	.021	-.034	-.033	.019	-.317[a]	–	
10. Math teachers' self-efficacy	-.039	.024	-.004	.003	.004	.009	.002	-.388[a]	.301[a]	–

[a] Correlation significant at 0.01 level (2-tailed).
[b] Correlation significant at 0.05 level (2-tailed).

SD = 0.03, $p < .001$; ICC = 0.20); the outcomes for the restricted model are therefore not presented in Table 3. In order to control for nesting within teacher/class, we finally computed the random effects for level 2 (class). Measures of children's PS development were thus corrected for the possible influences of teacher/class. Prior PS achievement was again the only significant predictor ($M = 0.74$, $SD = 0.03$, $p < .001$). This restricted model explained 31% of the total variance in the children's PS, T2 (ICC = 0.31).

3.3. *Teacher competencies as predictors of children's math development*

To examine how math development in grade four is predicted by teacher competencies, we conducted multi-level analyses that

Table 3
Children's math self-concept, math self-efficacy, and math anxiety as predictors of math development.

	Model 1 AF Unconditional	Model 1 PS Unconditional	Model 2 AF Level 1 Full model	Model 2 PS Level 1 Full model	Model 4 AF Level 2 (class)	Model 4 PS Level 2 (class)
Regression coefficients (fixed effects)						
Intercept	125.81** (1.52)	237.77*** (1.10)	44.61* (26.51)	76.26*** (13.70)	43.22*** (3.23)	76.96*** (6.63)
Prior achievement			0.77*** (0.28)	0.74*** (0.03)	0.78*** (0.03)	0.74*** (0.03)
Math self-concept			1.64** (0.53)	0.28 (0.46)	1.71*** (0.52)	0.43 (0.39)
Math self-efficacy			−0.88 (0.59)	0.32 (0.51)	−1.01 (0.54)	0.33 (0.44)
Math anxiety			0.15 (0.25)	0.17 (0.21)	0.30 (0.25)	0.17 (0.17)
Variance components (random effects)						
Intercept variance class					56.09 (0.00)	59.18 (40.17)
Prior achievement Variance					9.25 (0.00)	0.00 (0.00)
Math self-concept Variance					0.98 (0.54)	0.00 (0.00)
Math self-efficacy Variance					0.00 (0.00)	0.24 (0.29)
Math anxiety Variance					0.25 (0.36)	0.00 (0.00)
Variance part. ICC	0.10	0.26	0.11	0.22	0.14	0.31
−2 Log Likelihood	5210.83	5400.14	4458.58	4588.85	4395.38	4417.80

Note: $^{*}p < .05$, $^{**}p < .01$, $^{***}p < .001$.
AF = Arithmetic Fluency; PS = mathematical Problem-Solving.

examined actual teaching behavior, mathematical teaching knowledge, and math teachers' self-efficacy when measured at the start of the school year in relation to children's arithmetic fluency (AF, T1 and T2) and mathematical problem-solving (PS, T1 and T2).

For AF, we first computed the unconditional model (see Table 4 for the coefficients and ICCs). The unconditional model showed the level-1 AF scores of the children to vary significantly. The full model was next created by adding children's prior AF achievement and all of the teacher measures to the unconditional model as fixed effects. The full model showed a deviance statistic (−2 log likelihood) of 4517.05, indicating that the fit of the full model is significantly better than that of the null model ($\beta = 693.78$, $p < .001$). Children's prior AF achievement was, as might be expected, a significant predictor of their AF development ($M = 0.83$, $SD = 0.02$, $p < .001$). Actual teaching behavior was significantly but negatively related to AF development ($M = -11.34$, $SD = 3.66$, $p < .01$). Neither mathematical teaching knowledge related significantly to the development of AF ($M = -3.64$, $SD = 3.11$, $p = .24$) nor math teachers' self-efficacy ($M = 2.56$, $SD = 2.10$, $p = .23$).

When the restricted model was computed by removing all nonsignificant predictors of AF (in this case: mathematical teaching knowledge and math teachers' self-efficacy), a better fit was not obtained ($\beta = 38.30$, $SD = 7.73$, $p < .001$; prior AF achievement $M = 0.83$, $SD = 0.02$, $p < .001$; actual teaching behavior $M = -12.07$, $SD = 3.22$, $p < .001$; ICC = 0.10); the outcomes for this restricted model are therefore not included in Table 4. The level-1 full model still provides the best fit with the inclusion of children's prior AF achievement and measures of actual teaching behavior, mathematical teaching knowledge, and math teachers' self-efficacy together explaining 10% of the total variance in the children's AF (ICC = 0.10). In order to control for nesting within teacher/class, we finally computed the random effects for level 2 (class). The χ^2 change for this model including class variance with actual teaching behavior, mathematical teaching knowledge, and math teachers' self-efficacy was significant ($\chi^2 = 43.31$, $p < .001$). This model explained 11% of the total variance in the children's development AF (T1 and T2) (ICC = 0.11).

The same analyses were conducted to examine the influences of teacher competencies on the development of children's mathematical PS (see Table 4). The unconditional model showed the level-1 PS scores of the children to vary significantly. To create the full model, children's prior PS achievement and all three teacher measures were added to the unconditional model as fixed effects. The full model showed a deviance statistic (−2 log likelihood) of 4632.60, indicating a significantly better fit for the full model ($\beta = 767.54$, $p < .001$). As could be expected, the children's prior PS achievement significantly predicted their later PS achievement ($M = 0.78$, $SD = 0.03$, $p < .001$). In addition, all three teacher measures showed significant connections to children's math development (PS): actual teaching behavior was negatively related ($M = -10.65$, $SD = 3.02$, $p < .001$); mathematical teaching knowledge was positively related ($M = 8.85$, $SD = 2.55$, $p < .001$); and math teachers' self-efficacy was negatively related to children's later mathematical PS ($M = -5.29$, $SD = 1.70$, $p < .01$). This level-1 full model with the children's prior PS achievement included together with all of the teacher measures explained 21% of the total variance in the children's math development (i.e., mathematical PS, T1 and T2) (ICC = 0.21). The computation of a restricted model was not necessary.

Finally, we computed the random effects for level 2 (class) in order to control for nesting within classes for PS. This model showed a deviance statistic (−2 log likelihood) of 4479.27, which indicates added value. The χ^2 change proved significant for this model taking variance due to teacher/class into account ($\chi^2 = 153.33$, $p < .001$). The nested model including actual teaching behavior, mathematical teaching knowledge, and math teachers' self-efficacy explains 27% of the total variance in the children's development PS (ICC = 0.27).

3.4. Child and teacher factors as predictors of children's math development

We computed multilevel models to examine the influences of all of the child and teacher factors considered together on the children's fourth-grade math development. For arithmetic fluency (AF), we started with an unconditional model and found the level-1 AF scores of the children to vary significantly (Table 5). When we calculated the full prediction model, a deviance statistic (−2 log likelihood) of 4429.68 was found, showing the full model to fit significantly better than the unconditional model ($\beta = 644.26$, $p < .001$). This level-1 full model — containing all child and teacher factors — explained 11% of the total variance in the development of AF (T1, T2) (ICC = 0.11). We computed a restricted model by removing all nonsignificant predictors from the full model; only

Table 4
Teacher competencies as predictors of math development.

	Model 1 AF Unconditional	Model 1 PS Unconditional	Model 2 AF Level 1 Full model	Model 2 PS Level 1 Full model	Model 4 AF Level 2 (class)	Model 4 PS Level 2 (class)
Regression coefficients (fixed effects)						
Intercept	125.81*** (1.52)	237.77*** (1.10)	38.41*** (13.70)	68.52 *** (5.83)	38.08*** (2.79)	66.36*** (5.33)
Prior achievement			0.83*** (0.02)	0.78*** (0.03)	0.83*** (0.02)	0.79*** (0.02)
Actual teaching behavior			−11.34 ** (3.66)	−10.65*** (3.02)	−14.98* (7.28)	−13.85 (9.12)
Math. teaching knowledge			−3.64 (3.11)	8.85*** (2.55)	−6.43 (6.45)	0.57 (6.02)
Math teachers' self-efficacy			2.56 (2.10)	−5.29 ** (1.70)	0.22 (4.24)	−3.66 (4.12)
Variance components (random effects)						
Intercept variance class					36.85 (24.29)	0.00 (0.00)
Prior achievement					0.00 (0.00)	0.00 (0.00)
Actual teaching behavior Variance					0.00 (0.00)	916.79 (545.56)
Math. teaching knowledge Variance					0.00 (0.00)	0.00 (0.00)
Math teachers' self-efficacy Variance					0.00 (0.00)	0.00 (0.00)
Variance part. ICC	0.10	0.26	0.10	0.21	0.11	0.27
−2 Log Likelihood	5210.83	5400.14	4517.05	4632.60	4473.74	4479.27

Note: $^{*}p < .05$, $^{**}p < .01$, $^{***}p < .001$.
AF = Arithmetic Fluency; PS = mathematical Problem-Solving.

Table 5
Children's Math Self-concept, Math Self-efficacy, and Math Anxiety together with Teacher Competencies as Predictors of Children's Math Development.

	Model 1 AF Unconditional	Model 1 PS Unconditional	Model 2 AF Level 1 Full model	Model 2 PS Level 1 Full model	Model 4 AF Level 2 (class)	Model 4 PS Level 2 (class)
Regression coefficients (fixed effects)						
Intercept	125.81*** (1.52)	237.77*** (1.10)	44.09 *** (7.61)	75.18*** (7.04)	42.49*** (3.38)	81.22*** (7.08)
Prior achievement			0.77*** (0.28)	0.75*** (0.03)		
Math self-concept			1.66** (0.52)	0.20 (0.45)		
Math self-efficacy			−0.88 (0.59)	0.48 (0.50)		
Math anxiety			0.21 (0.24)	0.21 (0.20)		
Actual teaching behavior			−11.83*** (3.64)	−11.11*** (3.03)		
Math. teaching knowledge			−3.48 (3.08)	8.85** (2.55)		
Math teachers' self-efficacy			2.36 (2.09)	−5.42** (1.70)		
Variance components (random effects)						
Intercept variance class					0.00 (0.00)	63.08 (378.19)
Prior achievement Variance					0.63*** (0.17)	0.57*** (0.15)
Math self-concept Variance					1.63* (0.77)	0.16 (0.43)
Math self-efficacy Variance					0.00 (0.00)	0.62 (0.64)
Math anxiety Variance					0.06 (0.33)	0.00 (0.00)
Actual teaching behavior Variance					829.78 (1154.25)	10296.21 (7334.59)
Math. teaching knowledge Variance					0.00 (0.00)	0.00 (0.00)
Math teachers' self-efficacy Variance					363.09 (445.30)	0.00 (0.00)
Model summary						
Variance part. ICC	0.09	0.26	0.11	0.23	0.13	0.33
−2 Log Likelihood	5073.94	5085.16	4429.68	4545.89	4527.21	4577.08

Note: *$p < .05$, **$p < .01$, ***$p < .001$.
AF = Arithmetic Fluency; PS = mathematical Problem-Solving.

prior AF achievement, children's math self-concept, and actual teaching behavior remained in the restricted model. The level-1 restricted model did not provide a better fit ($\beta = 43.75$, $SD = 17.68$, $p < .001$; prior AF achievement $M = 0.78$, $SD = 0.03$, $p < .001$; children's math self-concept $M = 0.86$, $SD = 0.23$, $p < .001$; teachers' actual behavior $M = -12.39$, $SD = 3.19$, $p < .001$ (ICC = 0.11); the outcomes are therefore not included in Table 5. In order to control for nesting within teacher/class, we computed the random effects for level 2 (class). This model, in which children's AF development is corrected for the possible influences of teacher/class, provided the best fit (ICC = 0.13). Significant predictors were now prior AF achievement ($M = 0.63$, $SD = 0.17$, $p < .001$) and the children's math self-concept ($M = 1.63$, $SD = 0.77$, $p < .05$). Level-2 analyses showed an added class value of 2% relative to that for the full level-1 model.

Math development assessed in terms of mathematical problem-solving (PS) was analyzed next. In the initial unconditional model, the level-1 PS scores of the children were found to vary significantly (Table 5). For the full PS model, with all of the child and teacher factors included as fixed effects, a deviance statistic (−2 log likelihood) of 4545.89 was found, indicating that the full model provided a significantly better fit than the unconditional model ($\beta = 539.27$, $p < .001$). The full model — containing all child and teacher factors — explained 23% of the total variance in the children's PS (T1, T2) (ICC = 0.23).

We next computed a restricted model by removing all nonsignificant child and teacher factors from the full model; this meant removal math self-concept, math self-efficacy, and math anxiety for the children. This level-1 restricted model — now including all teacher factors in addition to the prior PS achievement of the children — did not provide a better fit than the full model ($\beta = 68.52$, $SD = 5.83$, $p < .001$; prior PS achievement $M = 0.78$, $SD = 0.03$, $p < .001$; teachers' actual behavior $M = -10.65$, $SD = 3.02$, $p < .001$; mathematical teaching knowledge $M = 8.85$, $SD = 2.55$, $p < .001$; self-efficacy M = - 5.28, $SD = 1.70$, $p < .01$ (ICC = 0.21). The results for the restricted model are therefore not included in Table 5. In order to control for nesting within teacher/class, we finally computed the random effects for level 2 (class). This nested model with children's PS math development corrected for the possible influences of teacher/class provided a better fit than just the level-1 full model (ICC = 0.33). The prior PS achievement of the children was now the only significant predictor ($M = 0.57$, $SD = 0.15$, $p < .001$). The level-2 analyses showed an added class value of 10% relative to that for the full level-1 model.

4.0. Discussion

In this study, we investigated longitudinally the prediction of the development of arithmetic fluency and mathematical problem-solving during the fourth grade for some 600 children. This was done on the basis of their math self-concept, math self-efficacy, and math anxiety but also the teacher competencies of actual teaching behavior, mathematical teaching knowledge, and math teachers' self-efficacy.

For the development of arithmetic fluency, both the children's arithmetic fluency at the start of fourth grade and their math self-concept were found to be significant positive predictors; actual teaching behavior was found to be a significant negative predictor.

For the development of mathematical problem-solving, both the children's mathematical problem-solving at the start of fourth grade and the teachers' mathematical knowledge were significant positive predictors; actual teaching behavior and math teachers' self-efficacy were significant negative predictors.

4.1. Child and teacher factors as predictors of math development

4.1.1. Child factors

We expected children's math self-concept, math self-efficacy, and math anxiety to predict the development of both children's arithmetic fluency and mathematical problem-solving ability in grade four. This expectation was tentative as previous studies typically involved older-aged children (e.g., McWilliams et al., 2013;

Pietsch et al., 2003; Timmerman et al., 2017) and produced inconsistent results. Out of the child factors, only math self-concept was found to be a significant predictor of arithmetic fluency in the present study, which aligns with the previous outcomes of Timmerman et al. (2017). Children's math self-concept is generally more past-oriented and stable than children's math self-efficacy, which — by definition — concerns the future (Möller et al., 2009). The influence of math self-concept on the development of arithmetic fluency, in particular, can therefore probably be explained by the fourth-grade children having greater experience with arithmetic than with mathematical problem-solving (Dweck, 2002; Marsh et al., 2005; Weidinger et al., 2018). In the lower elementary school grades, considerable attention is paid to basic arithmetic skills and understandably less attention to mathematical problem-solving.

We did not find children's math self-efficacy to significantly predict any of their math development, which is not consistent with the findings of older research (Pajares & Kranzler, 1995; Pietsch et al., 2003; Usher & Pajares, 2008, 2009). It is possible that math self-efficacy only predicts later development and thus development beyond fourth grade when children are better able to assess and align their expectations with regard to what they think that they can accomplish in specific math tasks (Pajares & Miller, 1994). In other words, elementary school children's self-efficacy within the domain of math is still malleable and can therefore be enhanced during their school careers — a possibility to be considered along with just how and when to do this in future research.

Math anxiety was also not found to be a significant predictor of any aspect of the children's math development. A possible explanation for this finding is that math anxiety has been found to generally increase during childhood (Dowker et al., 2016; Ma, 1999) and therefore probably not found to influence math development at the age of fourth grade children. An alternative explanation is that children in these schools experienced encouraging environments and thus developed positive math attitudes as a result (Beilock & Maloney, 2015).

The finding that children's mathematical problem-solving was not influenced in the present study by the children's math self-concept, math self-efficacy, or math anxiety is in contrast to the findings of previous research (Pajares & Kranzler, 1995; Ramirez et al., 2016). This led us to explore the results for low achievers in the present study, but the results of multilevel analyses showed no significant differences between this group of children and the total group of children.

4.1.2. Teacher factors

Just as for the child factors, we also found results contrary to what was expected for the influence of teacher factors on the children's fourth-grade math development. Although previous research has shown positive associations between actual teaching behavior and children's math achievement (e.g., Blazar, 2015; Reynolds & Muijs, 1999; Stronge et al., 2011; Van de Grift, 2007), we found only negative associations between actual math teaching behavior and the children's development (i.e., arithmetic fluency and mathematical problem-solving). This is in line with research that also found negative associations (Muijs & Reynolds, 2002).

This surprising negative influence of actual teaching behavior on children's math development might be due, at least in part, to the nature of elementary math education in the Netherlands (Hickendorff et al., 2017). Elementary math education in the Netherlands is characterized by a mixture of learning in contexts intended to encourage mathematical understanding and the practice of basic skills. Textbooks give teachers an important guideline for the identification and attainment of specific math goals. This teaching has been shown to start out well in the Netherlands (Hickendorff et al., 2017) but also call for a dynamic classroom context. Different math strengths, needs, and developmental pathways are encountered during elementary math teaching and call for additional teacher competencies, such as the ability to adapting math lessons and to conduct micro-interventions (Corno, 2008). Some teachers may simply not be able to respond effectively to the math needs of the children they are teaching. In older research, for example, Stipek, Givvin, Salmon, and MacGyvers (2001) found teachers to believe that they should fully control instruction and focus primarily on the acquisition of the skills, rules, and procedures needed to achieve correct performance rather than being focused on spontaneous learning, diverse thinking processes and mathematical understanding of children, which requires adaptive teacher competencies.

The teaching of mathematics is known to be complicated, involve longer-term learning processes, and indeed call for teachers to adapt their teaching to the different needs of the children in their classrooms (Ball, Thames, & Phelps, 2008; Corno, 2008). Muijs and Reynolds (2002) found that teachers perceive themselves to have more content knowledge and skills for teaching in the early mathematics domains compared to later domains of the mathematical curriculum (e.g., fractions and proportions). This suggests that teachers are aware of the importance of having sufficient mathematical teaching knowledge. With regard to the influence of the teachers' mathematical teaching knowledge, this was indeed found to be the case: it significantly predicted the development of the children's mathematical problem-solving in the present study. This finding is in line with the assumption that specific math competencies are required of teachers to teach and stimulate mathematical problem-solving (Kolovou, 2011; Walshaw & Anthony, 2008). Although teaching behavior that facilitates arithmetic fluency or mathematical problem-solving overlaps, specific accents are required. The development of arithmetic fluency requires teacher behavior that is aimed at the selection of appropriate problem-solving strategies in mathematics and practice with these strategies. This can generally be achieved using active, whole-class teaching (Kling & Bay-Williams, 2014; Muijs & Reynolds, 2000). In contrast, the development of mathematical problem-solving requires that the teacher pose think-activating questions, clearly verify solutions for children, be sensitive to the math needs of the children, flexible enough to meet the individual needs of children, and capable of checking that math goals have been achieved (Hiebert & Grouws, 2007; Van der Lans, Van de Grift, & Van Veen, 2018;, 2015; Verschaffel et al., 1999).

Finally and again contrary to what was expected on the basis of several previous studies (Ashton & Webb, 1986; Joët et al., 2011; Pietsch et al., 2003; Tella, 2008), the math teaching self-efficacy of the teachers *negatively* related to the development of the children's mathematical problem-solving and showed no significant associations with the development of their arithmetic fluency. These results suggest that the teacher's math self-efficacy may depend on the subdomain of math in question and whether, for example, they are being asked to stimulate arithmetic fluency or more abstract mathematical problem-solving. Teachers may not recognize the complexity of mathematical problem-solving for children and what the teaching of this requires. It is apparently difficult for teachers to identify what is necessary and apply this in more advanced math teaching situations.

According to Hiebert and Grouws (2007), a number of factors can hinder the development of effective math teaching behavior, such as a lack of not only subject matter knowledge but also the necessary pedagogical knowledge to teach math flexibly, and the absence of a useful knowledge base for teachers to improve their math teaching practices.

The Dunning-Kruger effect (Kruger & Dunning, 1999) might also be at play: less competent teachers fail to recognize their incompetency in teaching mathematical problem-solving. Self-assessment of math teaching self-efficacy by particularly teachers with a lower level of math teaching competence can actually lead to overestimation of their capacity to promote the development of mathematical problem-solving on the part of students.

In the models in which we combined child and teacher factors with control for the possible influences of teacher/class on math development, the results resembled those for the models in which child-related factors and teacher-related factors were distinguished.

4.2. Study strengths, limitations, and directions for further research

The present study involved a large sample of more than 500 children but a relatively small sample of 31 teachers. Caution is thus warranted when generalizing the results to other teachers.

First, we measured math self-concept, math-self-efficacy, and math anxiety in the manner used by others, namely by administration of a written self-perception questionnaire (e.g., Joët et al., 2011; McWilliams et al., 2013; Pajares & Kranzler, 1995; Ramirez et al., 2016). It is nevertheless possible that some of the fourth-grade children had difficulties responding to the questionnaires in writing their responses as opposed to other methods of measuring such as oral response on a questionnaire. One recent exception is a study by Viljaranta et al. (2014) in which a written self-concept scale was used in combination with the posing of a single question by an interviewer with fourth grade children and just a written questionnaire with seventh grade children. They still found children's math self-concept to not be predictive of subsequent math achievement. In addition, the limited number of questions used to address math self-concept, math self-efficacy, and math anxiety limit the generalizability of the present results. In future research, alternative means of measurement and using a greater number of questions, should be considered.

Second, the use of exclusively quantitative methods to assess both the teacher and child factors may not have fully captured the underlying character of the factors. Observational rating, for example, may not capture the richness of actual behavior during the teaching of a math lesson. Some examples of information that might have been missed are the exact nature of the questions posed by the teachers, the reaction of the teachers when the children adopt an approach that differs from the expected approach to solving a mathematical problem, or the use of specific math terminology by the teachers. The adoption of both quantitative and qualitative measures in the future might thus be fruitful (Lund, 2012). In such a manner and as recommended by Kyriakides et al. (2013), exactly what the teacher and the children do during a math lesson can be explored along with just how they interact. Another limitation to mention is that the outcome measure of teacher behavior is at the classroom level while our measures of the child factors are at the individual level.

Finally, observation of only a math lesson concerned with fractions and proportions may have limited our results. The teaching of various domains of math should thus be examined in the future and thereby allow us to compare the teaching of arithmetic fluency with the teaching of mathematical problem-solving. In line with the design of the present study, it is important in future research to recognize the possible specificity of the influences of various child and teacher factors depending on the particular domain of math teaching and math task being considered.

4.3. Implications for practice

The present results have shed light on the roles of various child and teacher factors in the math development of fourth-grade children. The findings have some clear implications for the practice of mathematics education.

First, prior math achievement was shown to contribute to both arithmetic fluency and mathematical problem-solving, which is in line with the findings of previous studies (Fuchs et al., 2006; Watts et al., 2014). Teachers should more clearly recognize the crucial role that they play in establishing a solid math base for elementary school children to build their further learning on. Teachers should be given a better understanding of exactly which aspects of their teaching are most effective for achieving given math learning goals and thereby making more informed decisions for the achievement of these learning goals (Hiebert & Grouws, 2007). A solid mathematical foundation in the lower elementary school grades or, in other words, early proficiency with numbers and numerical operations is a prerequisite for supplementing, refining, and deepening children's math knowledge, skill, and understanding (Byrnes & Wasik, 2009; Duncan et al., 2007).

Second, it is important to stimulate children's learning of new math concepts, the expansion of their math knowledge, and the mastery of more advanced math skills on the basis of prior learning and ability (National Research Council, 2001). Unfortunately, the best means to achieve these objectives are not completely clear. In any case, the results of the present study suggest that teachers must have not only sufficient mathematical knowledge but also sufficient pedagogical knowledge and math teaching self-efficacy to do this.

In addition, teachers should be encouraged as part of their professional development to attend more to the self-concepts of their students in general and their math self-concepts in particular. Once formed, negative self-perceptions can be very persistent (Swann, 2012). A clear association between children's math self-concept and arithmetic fluency was found in the present study, showing that it is crucial to provide the best opportunities for children to learn math early and feel confident about their math learning.

4.4. Conclusions

This study is one of the first to examine the joint influences of several child and teacher factors on children's math development over the course of a school year while distinguishing basic arithmetic fluency from more abstract mathematical problem-solving.

The findings support the assumption that children's math self-concept can clearly influence their math development and, in particular, the development of their arithmetic fluency in fourth grade. Children's prior math achievement was consistently the best predictor of their later math achievement in the various models tested by us. Establishment of a solid math foundation early in elementary school is thus critical for the subsequent development of children's math knowledge and skill.

As might be expected, the teachers' own math knowledge played an important role in the children's math development in the present study, in particular in the development of mathematical problem-solving. Actual teaching behavior during a math lesson, however, was *negatively* associated with the development of both the children's arithmetic fluency and mathematical problem-solving. In addition, the teachers' math teaching self-efficacy *negatively* related to the children's mathematical problem-solving. These unexpected results with regard to the influence of specific teacher competencies and self-perceptions on elementary school children's math development raise some intriguing questions about the classroom teaching of mathematics. How can teachers better attune their teaching to the math levels and needs of the children in their classrooms? How can teachers become more conscious of their math teaching behavior, enhance their math teaching competence, and become more confident about their math teaching in the end?

To summarize, the present study generated new knowledge for both the theory and practice of teaching elementary math. The results show the importance of promoting math self-confidence on the part of young children by giving them a solid math foundation for later learning. Further research on the influence of specific aspects of math teaching on specific aspects of children's math development is necessary to expand our knowledge of how we can best promote math development in both the early and later years of elementary school.

Declaration of conflicting interests

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Appendix A. Supplementary data

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